

CODE GENERATION

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These slides are motivated from Prof. Alex Aiken: Compilers (Stanford)

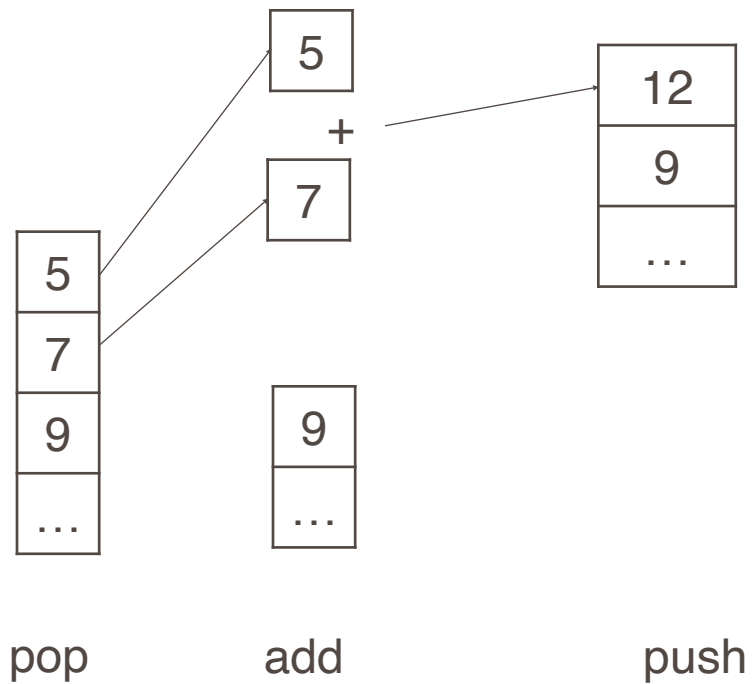


Stack Machine

- A simple evaluation model
- No variables or registers
- A stack of values for intermediate results
- Each instruction:
 - Takes its operands from the top of the stack
 - Removes those operands from the stack
 - Computes the required operation on them
 - Pushes the result on the stack

Example of Stack Machine Operation

- The addition operation on a stack machine



Example of a Stack Machine Program

- Consider two instructions
 - **push i** - place the integer i on top of the stack
 - **add** - pop two elements, add them and put the result back on the stack
- A program to compute $7 + 5$:

push 7

push 5

add

Why Use a Stack Machine?

- Each operation takes operands from the same place and puts results in the same place
- This means a uniform compilation scheme
- And therefore a simpler compiler

Why Use a Stack Machine?

- Location of the operands is implicit
 - Always on the top of the stack
- No need to specify operands explicitly
- No need to specify the location of the result
- Instruction “**add**” as opposed to “**add r1, r2**”
 - ⇒ Smaller encoding of instructions
 - ⇒ More compact programs
- This is one reason why Java Bytecodes use a stack evaluation model

Optimizing the Stack Machine

- The add instruction does 3 memory operations
 - Two reads and one write to the stack
 - The top of the stack is frequently accessed
- Idea: keep the top of the stack in a register (called accumulator)
 - Register accesses are faster
- The “add” instruction is now
$$\text{acc} \leftarrow \text{acc} + \text{top_of_stack}$$
 - Only one memory operation!

Stack Machine with Accumulator

- **Invariants**

- The result of an expression is in the accumulator
- For $op(e_1, \dots, e_n)$ push the accumulator on the stack after computing e_1, \dots, e_{n-1}
 - After the operation pops $n-1$ values
- **Expression evaluation preserves the stack**

Stack Machine with Accumulator. Example

- Compute $7 + 5$ using an accumulator
 1. $\text{acc} \leftarrow 7$; push acc
 2. $\text{acc} \leftarrow 5$
 3. $\text{acc} \leftarrow \text{acc} + \text{top_of_stack}$
 4. pop

A Bigger Example: $3 + (7 + 5)$

Code	ACC	Stack
acc ← 3	3	<init>
push acc	3	3,<init>
acc ← 7	7	3,<init>
push	7	7, 3,<init>
acc ← 5	5	7, 3,<init>
acc ← acc + top_of_stack	12	7, 3,<init>
pop	12	3,<init>
acc ← acc + top_of_stack	15	3,<init>
pop	15	<init>

It is very important evaluation of a subexpression preserves the stack

- Stack before the evaluation of $7 + 5$ is 3
- Stack after the evaluation of $7 + 5$ is 3
- The first operand is on top of the stack

From Stack Machines to MIPS

- The compiler generates code for a stack machine with accumulator
- Let's run the resulting code on a MIPS like processor.
 - Simulate stack machine instructions using MIPS instructions and registers
- The accumulator is kept in MIPS register $\$a0$
- The stack is kept in memory
 - The stack grows towards lower addresses
- The address of the next location on the stack is kept in MIPS register $\$sp$ (stack pointer)
 - The top of the stack is at address $\$sp + 4$

MIPS Assembly

- MIPS architecture
 - Prototypical Reduced Instruction Set Computer (RISC) architecture
 - Arithmetic operations use registers for operands and results
 - Must use load and store instructions to use operands and results in memory
 - 32 general purpose registers (32 bits each)
- We will use \$sp, \$a0 and \$t1 (a temporary register)

A Sample of MIPS Instructions

- **lw reg1 offset(reg2)**
 - Load 32-bit word from the value of reg2 (which is a memory address), add a fixed value offset into reg1
- **add reg1 reg2 reg3**
 - $\text{reg1} \leftarrow \text{reg2} + \text{reg3}$
- **sw reg1 offset(reg2)**
 - Store 32-bit word in reg1 at address $\text{reg2} + \text{offset}$
- **addiu reg1 reg2 imm**
 - $\text{reg1} \leftarrow \text{reg2} + \text{imm}$
 - “u” means overflow is not checked
- **li reg imm**
 - $\text{reg} \leftarrow \text{imm}$

MIPS Assembly, Example

- The stack-machine code for $7 + 5$ in MIPS:

Steps	MIPS Instruction
<code>acc = 7</code>	<code>li \$a0 7</code>
<code>push acc</code>	<code>sw \$a0 0(\$sp)</code> <code>addiu \$sp \$sp -4</code>
<code>acc ← 5</code>	<code>li \$a0 5</code>
<code>acc ← acc + top_of_stack</code>	<code>lw \$t1 4(\$sp)</code> <code>add \$a0 \$a0 \$t1</code>
<code>pop</code>	<code>addiu \$sp \$sp 4</code>

- Let's generalize this to a simple language

A Small Language

- A language with integers and integer operations

$P \rightarrow D; P \mid D$

$D \rightarrow \text{def id}(\text{ARGS}) = E;$

$\text{ARGS} \rightarrow \text{id}, \text{ARGS} \mid \text{id}$

$E \rightarrow \text{int} \mid \text{id} \mid \text{if } E_1 = E_2 \text{ then } E_3 \text{ else } E_4$
 $\mid E_1 + E_2 \mid E_1 - E_2 \mid \text{id}(E_1, \dots, E_n)$

- The first function definition f is the “main” routine
- Running the program on input i means computing $f(i)$
- Program for computing the Fibonacci numbers:

```
def fib(x) = if x = 1 then 0 else
             if x = 2 then 1 else
             fib(x - 1) + fib(x - 2)
```

Code Generation Strategy

- For each expression e we generate MIPS code that:
 - Computes the value of e in $\$a0$
 - Preserves $\$sp$ and the contents of the stack •
- We define a code generation function $cgen(e)$ whose result is the code generated for e
- The code to evaluate a constant simply copies it into the accumulator:

$$cgen(i) = li \$a0 i$$

- This preserves the stack, as required
- Color key:
 - RED: compile time
 - BLUE: run time

Code Generation for Add

```
cgen(e1 + e2) =  
    cgen(e1)  
    sw $a0 0($sp)  
    addiu $sp $sp -4  
    cgen(e2)  
    lw $t1 4($sp)  
    add $a0 $t1 $a0  
    addiu $sp $sp 4
```

Code Generation for Add

`cgen(e1 + e2) =`

`cgen(e1)`

`sw $a0 0($sp)`

`addiu $sp $sp -4`

`cgen(e2)`

`lw $t1 4($sp)`

`add $a0 $t1 $a0`

`addiu $sp $sp 4`

`cgen(e1 + e2) =`

`cgen(e1)`

`print "sw $a0 0($sp)"`

`print "addiu $sp $sp -4"`

`cgen(e2)`

`print "lw $t1 4($sp)"`

`print "add $a0 $t1 $a0"`

`print "addiu $sp $sp 4"`

Code Generation for Add. Wrong!

- Optimization: Put the result of e_1 directly in $\$t1$?

`cgen(e1 + e2) =`

`cgen(e1)`

`move $t1 $a0`

`cgen(e2)`

`add $a0 $t1 $a0`

- Try to generate code for : $3 + (7 + 5)$

Code Generation Notes

- The code for $+$ is a template with “holes” for code for evaluating e_1 and e_2
- Stack machine code generation is recursive
 - Code for $e_1 + e_2$ is code for e_1 and e_2 glued together
- Code generation can be written as a recursive descent of the AST
 - At least for expressions

Code Generation for Sub and Constants

- New instruction: `sub reg1 reg2 reg3`

Implements `reg1 ← reg2 - reg3`

`cgen(e1 - e2) = cgen(e1)`

`sw $a0 0($sp)`

`addiu $sp $sp -4`

`cgen(e2)`

`lw $t1 4($sp)`

`sub $a0 $t1 $a0`

`addiu $sp $sp 4`

From what expression the following assembly code is generated?

```
li $a0 5
sw $a0 0($sp)
addiu $sp $sp -4
li $a0 4
sw $a0 0($sp)
addiu $sp $sp -4
li $a0 3
lw $t1 4($sp)
sub $a0 $t1 $a0
addiu $sp $sp 4
lw $t1 4($sp)
add $a0 $t1 $a0
addiu $sp $sp 4
```

Code Generation for Conditional

- We need flow control instructions
- New instruction: `beq reg1 reg2 label`
 - Branch to label if `reg1 = reg2`
- New instruction: `b label`
 - Unconditional jump to label

Code Generation for If (Cont.)

```
cgen(if e1 = e2 then e3 else e4) =  
    cgen(e1)  
    sw $a0 0($sp)  
    addiu $sp $sp -4  
    cgen(e2)  
    lw $t1 4($sp)  
    addiu $sp $sp 4  
    beq $a0 $t1 true_branch  
  
false_branch:  
    cgen(e4)  
    b end_if  
true_branch:  
    cgen(e3)  
end_if:
```


The Activation Record

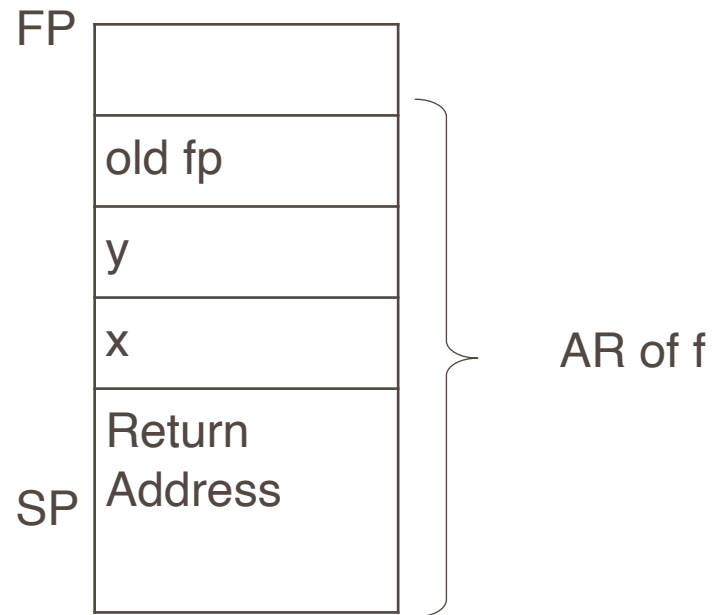
- Code for function calls and function definitions depends on the layout of the AR
- A very simple AR suffices for this language:
 - The result is always in the accumulator
 - No need to store the result in the AR
 - The activation record holds actual parameters
 - For $f(x_1, \dots, x_n)$ push x_n, \dots, x_1 on the stack
 - These are the only variables in this language

The Activation Record (Cont.)

- The stack discipline guarantees that on function exit $\$sp$ is the same as it was on function entry
- We need the return address
- A pointer to the current activation is useful
 - This pointer lives in register $\$fp$ (frame pointer)
 - Reason for frame pointer will be clear shortly

The Activation Record

- Summary: For this language, an AR with the caller's frame pointer, the actual parameters, and the return address suffices
- Picture: Consider a call to $f(x,y)$, the AR is:



Code Generation for Function Call

- The calling sequence is the instructions (of both caller and callee) to set up a function invocation
- New instruction: `jal label`
 - Jump to label, save address of next instruction in `$ra`
 - On other architectures the return address is stored on the stack by the “call” instruction

Code Generation for Function Call (Cont.)

```
cgen(f(e1, ..., en)) =  
    sw $fp 0($sp)  
    addiu $sp $sp -4  
    cgen(en)  
    sw $a0 0($sp)  
    addiu $sp $sp -4  
    ...  
    cgen(e1)  
    sw $a0 0($sp)  
    addiu $sp $sp -4  
    jal f_entry
```

- The caller saves its value of the frame pointer
- Then it saves the actual parameters in reverse order
- The caller saves the return address in register `$ra`
- The AR so far is $4*n+4$ bytes long

Code Generation for Function Definition

- New instruction: `jr reg`
 - Jump to address in register `reg`

```
cgen(def f(x1,...,xn) = e) =  
fEntry:
```

```
  move $fp $sp  
  sw $ra 0($sp)  
  addiu $sp $sp -4  
  cgen(e)  
  lw $ra 4($sp)  
  addiu $sp $sp z  
  lw $fp 0($sp)  
  jr $ra
```

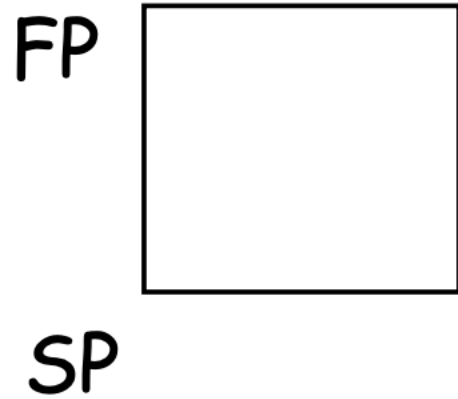
Note: The frame pointer points to the top,
not bottom of the frame

The callee pops the return address, the actual
arguments and the saved value of the frame pointer.

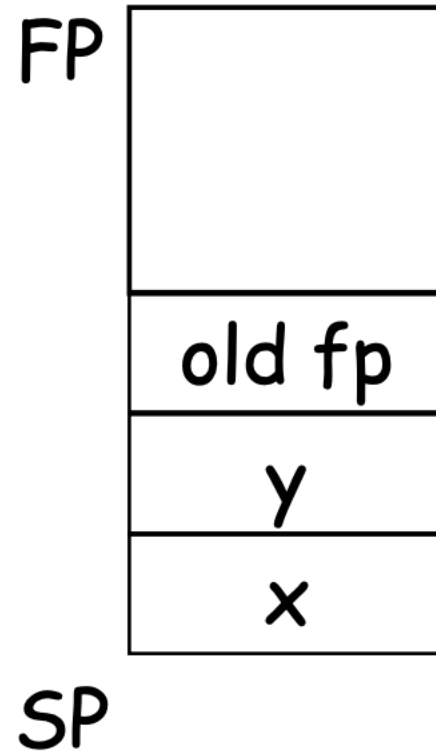
$$z = 4*n + 8$$

Calling Sequence: Example for $f(x,y)$

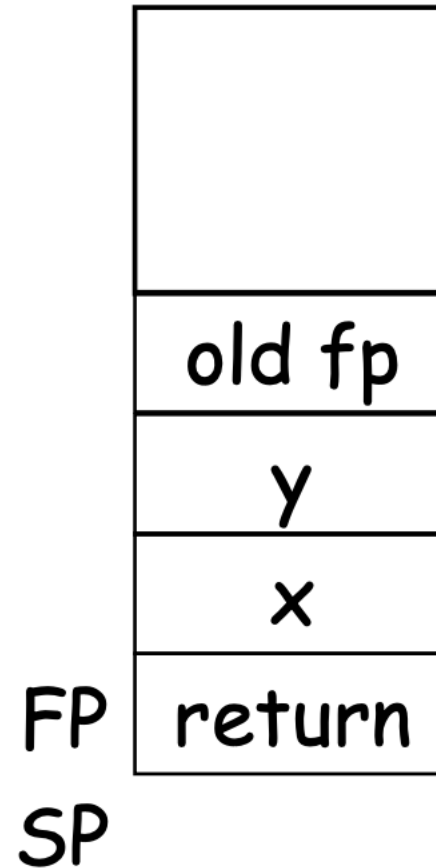
Before call



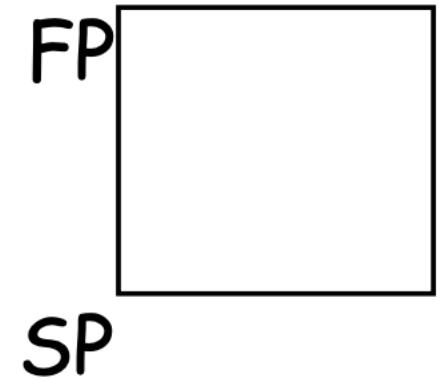
On entry



Before exit



After call



Code Generation for Variables

- Variable references are the last construct
- The “variables” of a function are just its parameters
 - They are all in the AR
 - Pushed by the caller
- Problem: Because the stack grows when intermediate results are saved, the variables are not at a fixed offset from `$sp`

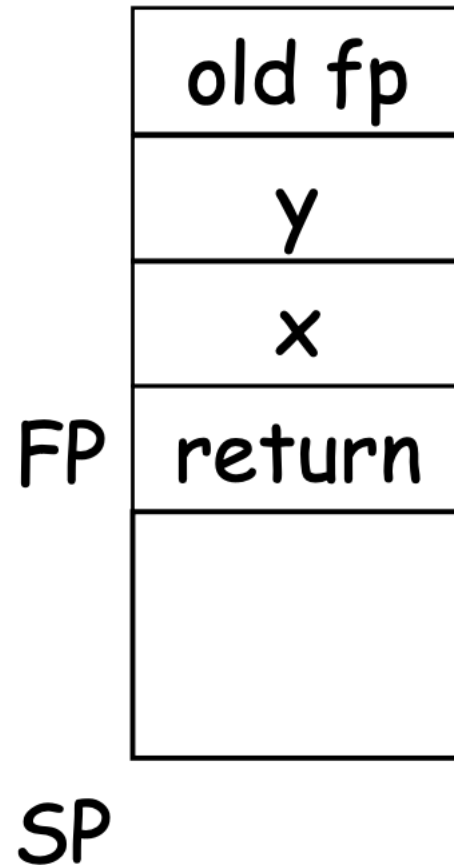
Code Generation for Variables (Cont.)

- Solution: use a frame pointer
 - Always points to the return address on the stack
 - Since it does not move it can be used to find the variables
- Let x_i be the i^{th} ($i = 1, \dots, n$) formal parameter of the function for which code is being generated

$\text{cgen}(x_i) = \text{lw } \$a0 \text{ } z(\$fp) \text{ (} z = 4*i \text{)}$

Code Generation for Variables (Cont.)

- Example: For a function `def f(x,y) = e` the activation and frame pointer are set up as follows:



- X is at `fp + 4`
- Y is at `fp + 8`

Summary

- The activation record must be designed together with the code generator.
- Code generation can be done by recursive traversal of the AST.
- Production compilers do different things
 - Emphasis is on keeping values (esp. current stack frame) in registers
 - Intermediate results are laid out in the AR, not pushed and popped from the stack

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

cumsumEntry:

```
move $fp $sp
sw $ra 0($sp)
addiu $sp $sp -4
lw $a0 4($fp)
sw $a0 0($sp)
addiu $sp $sp -4
li $a0 0
lw $t1 4($sp)
addiu $sp $sp 4
beq $a0 $t1 true_branch
```

false_branch:

```
lw $a0 4($fp)
sw $a0 0($sp)
addiu $sp $sp -4
sw $fp 0($sp)
addiu $sp $sp -4
lw $a0 4($fp)
sw $a0 0($sp)
addiu $sp $sp -4
li $a0 1
lw $t1 4($sp)
sub $a0 $t1 $a0
addiu $sp $sp 4
```

```
sw $a0 0($sp)
addiu $sp $sp -4
jal cumsumEntry
lw $t1 4($sp)
add $a0 $t1 $a0
addiu $sp $sp 4
b endif1
```

true_branch:

```
li $a0 0
```

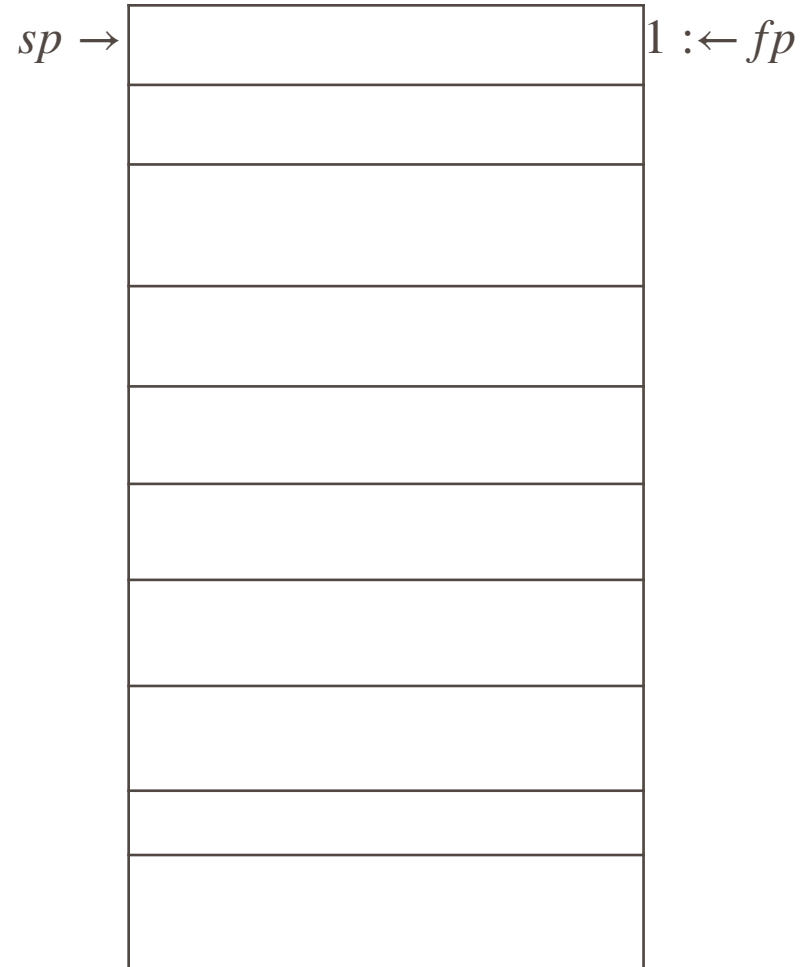
endif1:

```
lw $ra 4($sp)
addiu $sp $sp 12
lw $fp 0($sp)
jr $ra
```


Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

cumsumEntry:

1. **move \$fp \$sp**
2. **sw \$ra 0(\$sp)**
3. **addiu \$sp \$sp -4**
4. **lw \$a0 4(\$fp)**
5. **sw \$a0 0(\$sp)**
6. **addiu \$sp \$sp -4**
7. **li \$a0 0**
8. **lw \$t1 4(\$sp)**
9. **addiu \$sp \$sp 4**
10. **beq \$a0 \$t1 true_branch**



Example: $\text{def cumsum}(x) = \text{if } x = 0 \text{ then } 0 \text{ else } x + \text{cumsum}(x-1)$

cumsumEntry:

1. `move $fp $sp`
2. `sw $ra 0($sp)`
3. `addiu $sp $sp -4`
4. `lw $a0 4($fp)`
5. `sw $a0 0($sp)`
6. `addiu $sp $sp -4`
7. `li $a0 0`
8. `lw $t1 4($sp)`
9. `addiu $sp $sp 4`
10. `beq $a0 $t1 true_branch`

6 : *sp* →

2: ra
5: x

1 : $\leftarrow fp$

4 : $a0 = 4 + fp = x$

7 : $a0 = 0$

8 : $t1 = 4 + sp = x$

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

Overall

cumsumEntry:

1. move \$fp \$sp
2. sw \$ra 0(\$sp)
3. addiu \$sp \$sp -4
4. lw \$a0 4(\$fp)
5. sw \$a0 0(\$sp)
6. addiu \$sp \$sp -4
7. li \$a0 0
8. lw \$t1 4(\$sp)
9. addiu \$sp \$sp 4
10. beq \$a0 \$t1 true_branch

starting : sp →

9 : sp → 3 : sp →

6 : sp →

2. ra
5. x

1 : ← fp

4 : a0 = 4 + fp = x

7 : a0 = 0

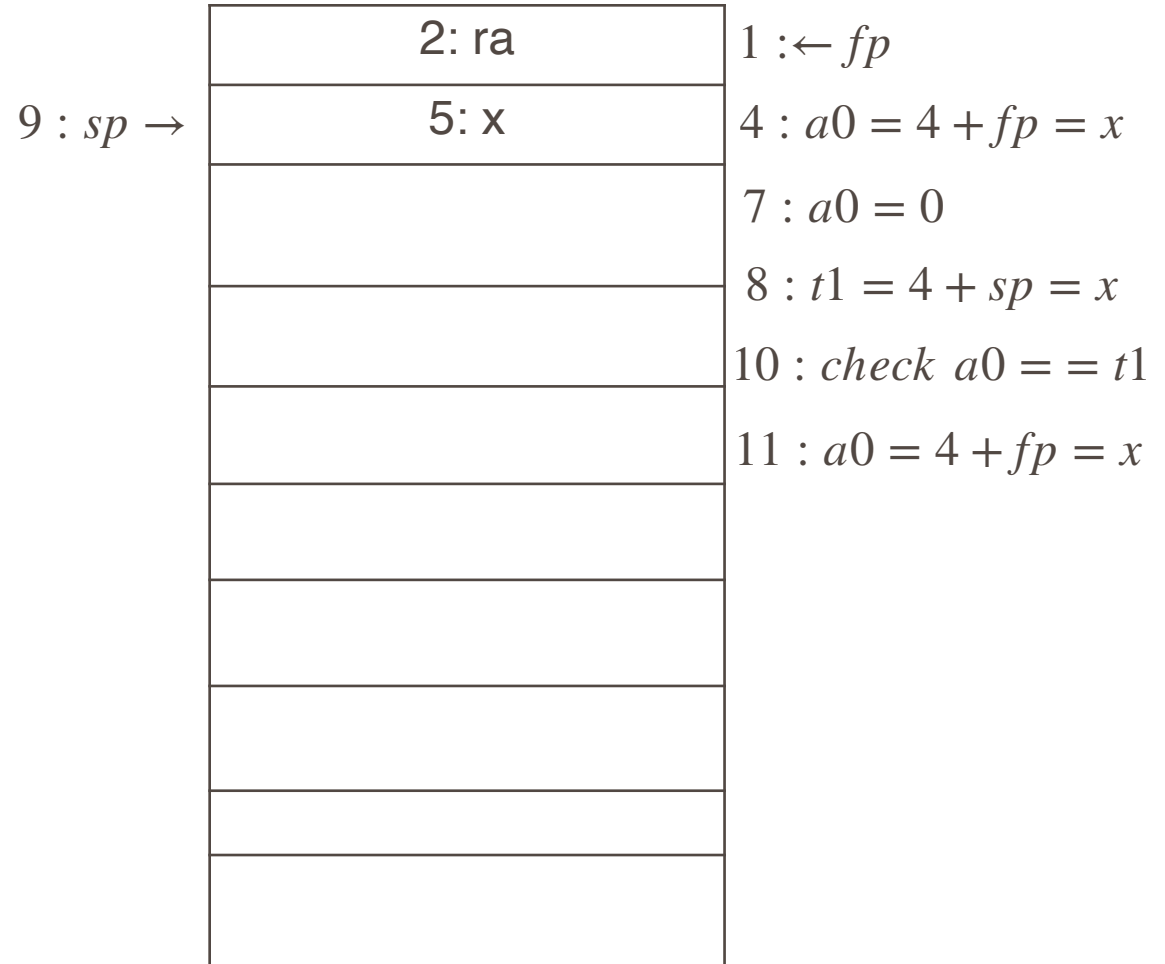
8 : t1 = 4 + sp = x

10 : check a0 == t1

Example: $\text{def cumsum}(x) = \text{if } x = 0 \text{ then } 0 \text{ else } x + \text{cumsum}(x-1)$

false_branch:

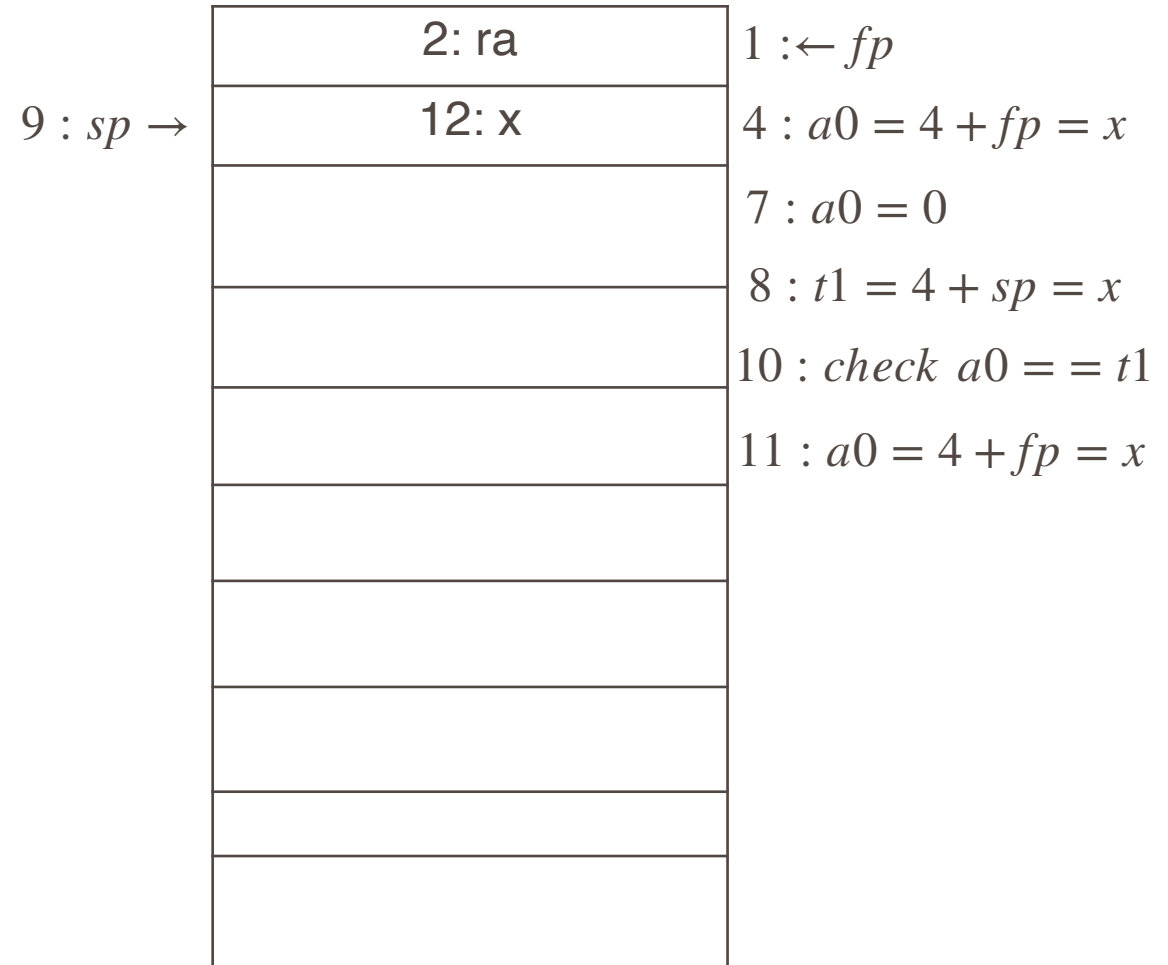
```
11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1
```



Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

false_branch:

```
11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1
```



Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

13 : *sp* →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14. fp (this is old <i>fp</i> of cumsumEntry())	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
	10 : <i>check</i> $a0 == t1$
	11 : $a0 = 4 + fp = x$

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

```

false_branch:
11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

15 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14. fp (this is old fp of cumsumEntry(x))	7 : a0 = 0
	8 : t1 = 4 + sp = x
	10 : check a0 == t1
	11 : a0 = 4 + fp = x

Example: $\text{def cumsum}(x) = \text{if } x = 0 \text{ then } 0 \text{ else } x + \text{cumsum}(x-1)$

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

15 : sp →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14. fp (this is old fp of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
	10 : <i>check</i> $a0 == t1$
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	16 : $a0 = 4 + fp = x$

Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

false_branch:

```

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16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

15 : *sp* →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14: fp (this is old <i>fp</i> of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
17: x	10 : <i>check</i> $a0 == t1$
	11 : $a0 = 4 + fp = x$
	16 : $a0 = 4 + fp = x$

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

18 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
17: x	10 : check a0 == t1
	11 : a0 = 4 + fp = x
	16 : a0 = 4 + fp = x

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

18 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
17: x	10 : check a0 == t1
	11 : a0 = 4 + fp = x
	16 : a0 = 4 + fp = x
	19 : a0 = 1

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

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12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

18 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
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false_branch:

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19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

18 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
17: x	10 : check a0 == t1
	11 : a0 = 4 + fp = x
	16 : a0 = 4 + fp = x
	19 : a0 = 1
	20 : t1 = 4 + sp = x
	21 : a0 = t1 - a0 = x - 1

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

22 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
17: x	10 : check a0 == t1
	11 : a0 = 4 + fp = x
	16 : a0 = 4 + fp = x
	19 : a0 = 1
	20 : t1 = 4 + sp = x
	21 : a0 = t1 - a0 = x - 1

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

22 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
23: x-1	10 : check a0 == t1 11 : a0 = 4 + fp = x 16 : a0 = 4 + fp = x 19 : a0 = 1 20 : t1 = 4 + sp = x 21 : a0 = t1 - a0 = x - 1

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

22 : sp →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14: fp (this is old fp of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
23: x-1	10 : <i>check</i> $a0 == t1$
	11 : $a0 = 4 + fp = x$
	16 : $a0 = 4 + fp = x$
	19 : $a0 = 1$
	20 : $t1 = 4 + sp = x$
	21 : $a0 = t1 - a0 = x - 1$

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

24 : sp →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14: fp (this is old fp of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
23: x-1	10 : <i>check</i> $a0 == t1$
	11 : $a0 = 4 + fp = x$
	16 : $a0 = 4 + fp = x$
	19 : $a0 = 1$
	20 : $t1 = 4 + sp = x$
	21 : $a0 = t1 - a0 = x - 1$

Example: $\text{def cumsum}(x) = \text{if } x = 0 \text{ then } 0 \text{ else } x + \text{cumsum}(x-1)$

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1
  
```

25 : sp →

2: ra
12: x
14: fp (this is old fp of cumsumEntry(x))
23: x-1

1 : $\leftarrow fp$
4 : $a0 = 4 + fp = x$
7 : $a0 = 0$
8 : $t1 = 4 + sp = x$
10 : check $a0 == t1$
11 : $a0 = 4 + fp = x$
16 : $a0 = 4 + fp = x$
19 : $a0 = 1$
20 : $t1 = 4 + sp = x$
21 : $a0 = t1 - a0 = x - 1$
25 : jal cumsumEntry
 (result stored in a0)
 (sp is reset to before call state)

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

25 : sp →

2: ra	1 : ← fp
12: x	4 : a0 = 4 + fp = x
14: fp (this is old fp of cumsumEntry(x))	7 : a0 = 0 8 : t1 = 4 + sp = x
23: x-1	10 : check a0 == t1
	11 : a0 = 4 + fp = x
	16 : a0 = 4 + fp = x
	19 : a0 = 1
	20 : t1 = 4 + sp = x
	21 : a0 = t1 - a0 = x - 1
	25 : jal cumsumEntry (result stored in a0) (sp is reset to before call state)
	26 : t1 = 4 + fp = x

Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1
    
```

25 : *sp* →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14: fp (this is old fp of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
23: x-1	10 : check $a0 == t1$ 11 : $a0 = 4 + fp = x$ 16 : $a0 = 4 + fp = x$
	19 : $a0 = 1$ 20 : $t1 = 4 + sp = x$
	21 : $a0 = t1 - a0 = x - 1$ 25 : <i>jal cumsumEntry</i> (result stored in a0) (sp is reset to before call state)
	26 : $t1 = 4 + fp = x$

27 : $a0 = t1 + a0 = x + cumsum(x - 1)$

Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

28 : *sp* →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14: fp (this is old fp of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
23: x-1	10 : <i>check</i> $a0 == t1$
	11 : $a0 = 4 + fp = x$
	16 : $a0 = 4 + fp = x$
	19 : $a0 = 1$
	20 : $t1 = 4 + sp = x$
	21 : $a0 = t1 - a0 = x - 1$
	25 : <i>jal cumsumEntry</i> (result stored in a0) (sp is reset to before call state)
	26 : $t1 = 4 + fp = x$

27 : $a0 = t1 + a0 = x + cumsum(x - 1)$

Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

false_branch:

```

11. lw $a0 4($fp)
12. sw $a0 0($sp)
13. addiu $sp $sp -4
14. sw $fp 0($sp)
15. addiu $sp $sp -4
16. lw $a0 4($fp)
17. sw $a0 0($sp)
18. addiu $sp $sp -4
19. li $a0 1
20. lw $t1 4($sp)
21. sub $a0 $t1 $a0
22. addiu $sp $sp 4
23. sw $a0 0($sp)
24. addiu $sp $sp -4
25. jal cumsumEntry
26. lw $t1 4($sp)
27. add $a0 $t1 $a0
28. addiu $sp $sp 4
29. b endif1

```

28 : *sp* →

2: ra	1 : $\leftarrow fp$
12: x	4 : $a0 = 4 + fp = x$
14: fp (this is old fp of cumsumEntry(x))	7 : $a0 = 0$ 8 : $t1 = 4 + sp = x$
23: x-1	10 : <i>check</i> $a0 == t1$
	11 : $a0 = 4 + fp = x$
	16 : $a0 = 4 + fp = x$
	19 : $a0 = 1$
	20 : $t1 = 4 + sp = x$
	21 : $a0 = t1 - a0 = x - 1$
	25 : <i>jal cumsumEntry</i> (result stored in a0) (sp is reset to before call state)
	26 : $t1 = 4 + fp = x$

27 : $a0 = t1 + a0 = x + cumsum(x - 1)$

Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

```
true_branch:  
30.  li $a0 0
```

28 : *sp* →

2: ra
12: x
14: fp (this is old <i>fp</i> of cumsumEntry(x))
23: x-1

1 : ← *fp*

30 : *a0* = 0

Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

```
/* at this point a0 either  
contains 0 from true branch  
or x + cumsum(x-1) from  
false branch */
```

```
endif1:
```

```
31. lw $ra 4($sp)  
32. addiu $sp $sp 12  
33. lw $fp 0($sp)  
34. jr $ra
```

28 : sp →

2: ra
12: x
14: fp (this is old fp of cumsumEntry(x))
23: x-1

1 :← fp

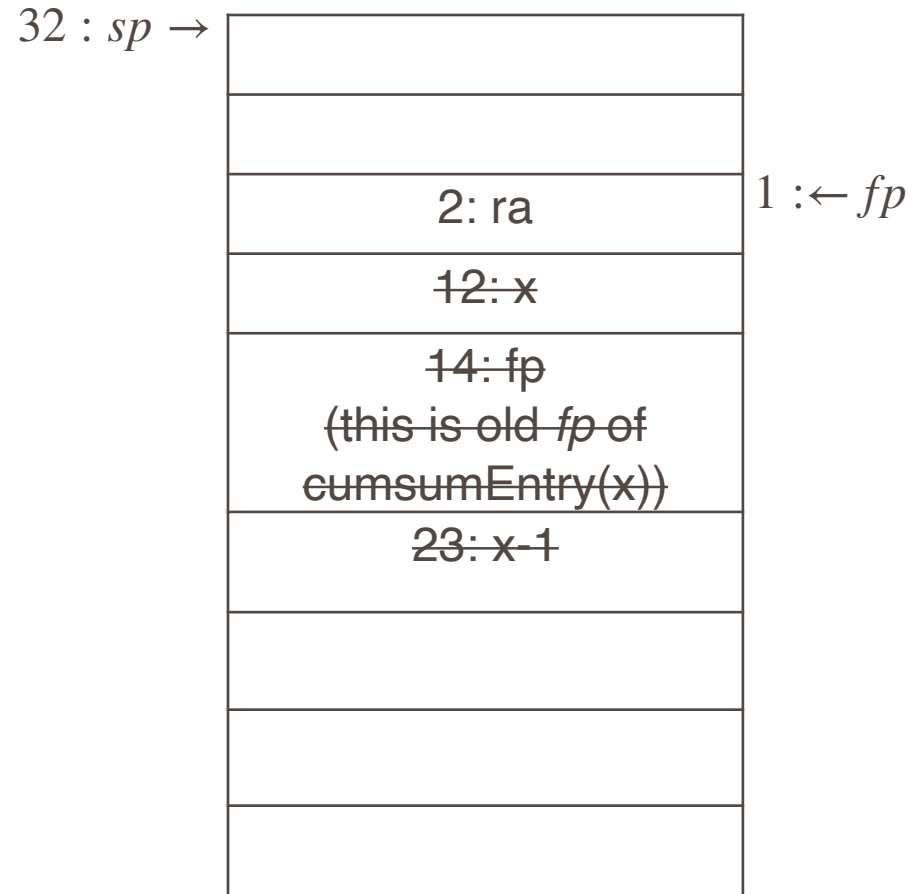
31 : ra = ra

Example: $\text{def cumsum}(x) = \text{if } x = 0 \text{ then } 0 \text{ else } x + \text{cumsum}(x-1)$

/* at this point a0 either
contains 0 from true branch
or $x + \text{cumsum}(x-1)$ from
false branch */

endif1:

```
31. lw $ra 4($sp)
32. addiu $sp $sp 12
33. lw $fp 0($sp)
34. jr $ra
```

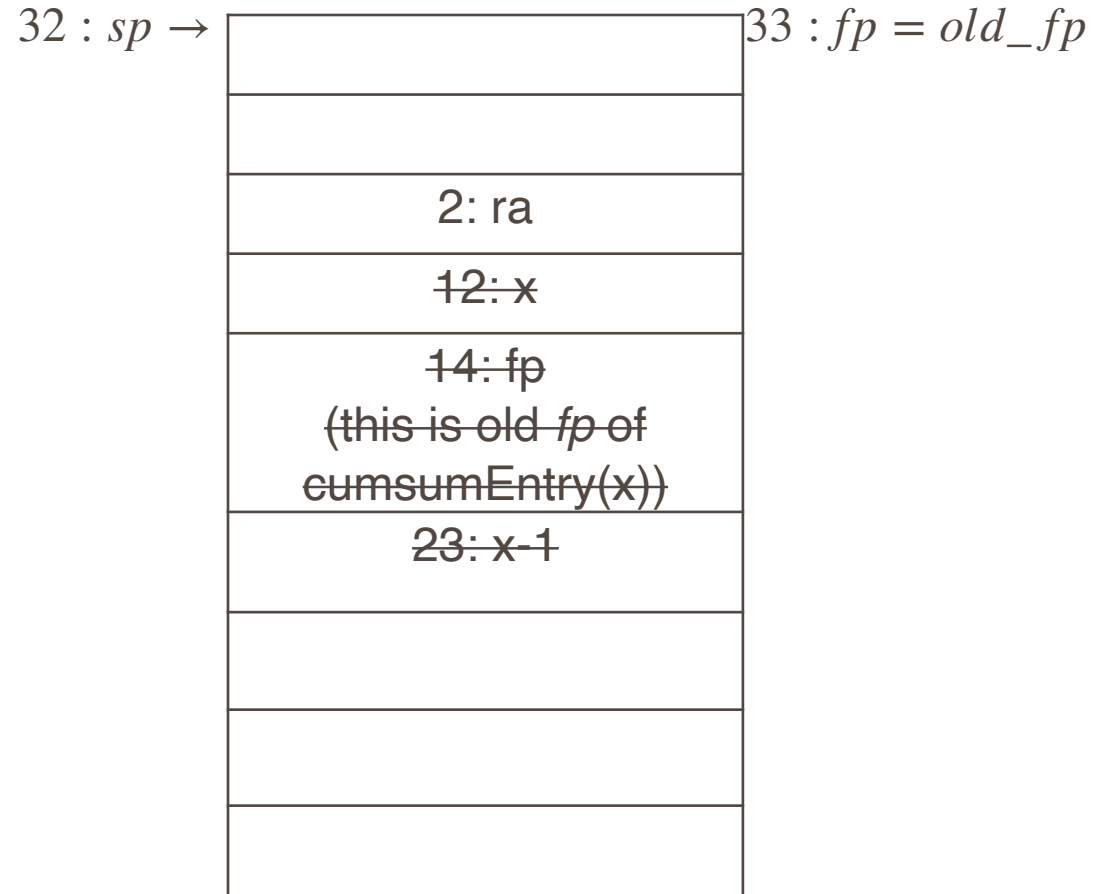


Example: `def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)`

```
/* at this point a0 either
contains 0 from true branch
or x + cumsum(x-1) from
false branch */
```

```
endif1:
```

```
31. lw $ra 4($sp)
32. addiu $sp $sp 12
33. lw $fp 0($sp)
34. jr $ra
```



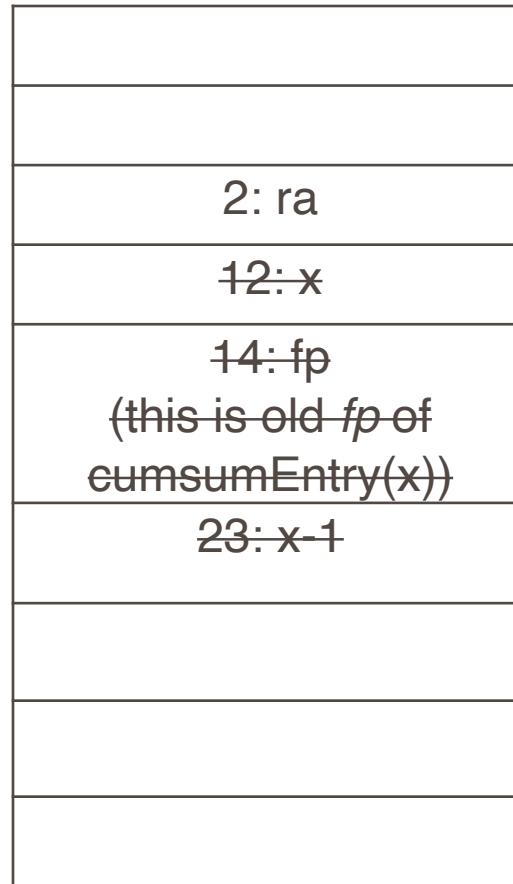
Example: def cumsum(x) = if x = 0 then 0 else x + cumsum(x-1)

```
/* at this point a0 either  
contains 0 from true branch  
or x + cumsum(x-1) from  
false branch */
```

```
endif1:
```

```
31. lw $ra 4($sp)  
32. addiu $sp $sp 12  
33. lw $fp 0($sp)  
34. jr $ra
```

32 : *sp* →



33 : *fp* = *old_fp*

34 : *jump to return address stored in ra*